Network Pollution Games*

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ABSTRACT

We introduce a new network model of the pollution control problem and present two applications of this model. On a high level, our model comprises a graph whose nodes represent the agents, that could be thought of as sources of pollution, and edges between agents represent the effect of spread of pollution. The government as the regulator is responsible to maximize the social welfare while setting bounds on the levels of emitted pollution both locally and globally. Our model is inspired by the existing literature in environmental economics that applies game theoretical methodology to control pollution. We study the social welfare maximization problem in our model. Our main results include hardness results for the problem, and in complement, a constant approximation algorithm on planar graphs. Our approximation algorithm leads to a truthful in expectation mechanism, and it is obtained by a novel decomposition technique of planar graphs to deal with constraints on vertices. We note that no known planar decomposition techniques can be used here and our technique can be of independent interest.

Keywords

Algorithmic mechanism design; approximation algorithms; planar graphs; pollution control

1. INTRODUCTION

Technology advance and commercial freedom have fused and accelerated the development process in an unprecedented scale. Environmental degradation however has accompanied this progress, resulting in global water and air pollution. In many developing countries, this has caused wide public concerns. As an example, in 2012, China discharged 68.5 billion tons of industrial wastewater, and the SO_2 emissions reached 21.2 million tons (National Bureau of Statistics of China, 2013). China has become one of the most polluted countries in the world, with industrial emissions as the main source of its pollution. The recent annual State of the Air report of the American Lung Association finds 47% of Americans live in counties with frequently unhealthy levels of either ozone or particulate pollution, see [4]. The latest assessment of air quality, by the European Environment Agency, finds that around 90% of city inhabitants in the European Union are exposed to one of the most damaging air pollutants at harmful levels, see [1]. It is the role of regulatory authorities to make efficient environmental policies in balancing economic growth and environment protection.

Our model and motivation. Pollution control regulations are inspired by the managerial approaches in environment policies, where models based on game theory are proposed and analysed. Kwerel [25] proposed a mechanism where firms, potential polluters, report their clean-up cost information to the regulator. The regulator sells a fixed number of pollution licences at a fixed price per licence and offers a subsidy for those licences which firms hold in excess of emission, based on the cost information provided by firms. In Kwerel's mechanism truth-telling by all firms implies a Nash equilibrium. Kwerel's scheme maintains a mild level of pollution by optimizing the social welfare (sum of the global clean-up cost and damage cost of emitted pollution).

From a different viewpoint, Dasgupta, Hammond and Maskin [11] focus on minimizing the sum of pollution damages, abatement costs and individual rationality for consumers. Spulber [36] develops a market model of environmental regulation with interdependent production, pollution abatement costs and heterogeneous firms who have private information about costs and pursue Bayes-Nash strategies in communication with the regulator. Their paper illustrates that the full information optimum cannot be attained unless gains from trade in the product market net of external damages exceed the information rents earned by firms, and aggregate output and externality levels are lower at the regulated equilibrium than at the full information social optimum.

Pollution has a diffusion nature: emitted at one source, it will have an effect on its neighbours at some diminish-

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ing level. We consider two applications using a network model. In the first application, the vertices represent pollution sources and edges are routes of pollution transition from one source to another similar to Belitskaya [9]. Our model measures the pollution diminishing transition by arbitrary weights on the edges, which is also present in the model by Montgomery [28]. The polluters' privately known clean-up cost and damage of the emitted pollution in our model are inspired by Kwerel. In the second application, the vertices represent mayors of cities and edges the roads between cities. The percentage of cars moving from one city to another is represented by the weight of the corresponding edge.

Our model covers both aforementioned applications with details given in section 3. The government as the regulator can decide to either shut down or keep open a pollution source taking into account the diffusion nature of pollution. It sets bounds on global and local levels of pollution, while trying to optimize the social welfare. The emissions that exceed the licences, if any, must be cleaned-up (hence, agent's clean-up cost). Furthermore our model allows the regulator to auction pollution licences for cars to mayors. In this case, the pollution level of an agent (mayor), i.e., the number of allocated licences, is set by the regulator together with the prices that the agent pays to get them.

Finding an optimal social welfare maximizing solution to our problem, which we call Pollution Game (PG), is NP-hard, that is why we study polynomial time approximation algorithms which can lead to incentive compatible (truthful) mechanisms. We study linear cost and damage functions, and derive approximation algorithms and truthful mechanisms focusing on planar network topologies. In contrast, Belitskaya assumes quadratic cost functions and linear damage functions and derives optimal social welfare and Nash equilibria solutions by explicit analytic formulas. We focus our study on planar network topologies which model realistic scenarios.

Most of the cited economics papers derive equilibria by closed analytic formulas. Some of these papers provide computational mechanisms without investigating polynomial running time. Our approach is algorithmic and focuses on efficiently computing these solutions. We also analyse the computational complexity/hardness, of computing the social optimum in our model. To the best of our knowledge, our work is the first attempt to algorithmically analyze pollution control from the perspective of regulators by a network game model with information asymmetry between regulators and polluters.

Technical contributions. Our main algorithmic contributions are for linear objective functions on planar graphs. Baker's shifting and tree-width decomposition techniques, see, e.g., [6, 21], are used for designing PTASs [1] for various problems on planar graphs. It seems impossible to design a PTAS for PG with binary variables on planar graphs by adapting these techniques. That is because they deal with constraints on edges (e.g., for the independent set problem), but PG's constraints are imposed on vertices from its neighbouring vertices. More precisely, given two optimal solutions on two subgraphs (with common boundary vertices) of the planar graph, combining them together may not result in a feasible solution for PG on the whole graph.This is due to the possible infeasibility of local constraints of the boundary vertices of these two subgraphs. We overcome this major difficulty by introducing a new decomposition technique of planar graphs, an (α, β)-decomposition. This new technique is of independent interest and it may have further applications for the problems with constraints on vertices rather than on edges.

Even when polluters' cost functions are linear with a single parameter, simple monotonicity is not sufficient to turn our algorithms into truthful mechanisms (see e.g. Chapter 11 in [29]). This is because polluters' utility functions have externalities – they are affected by their neighbours. Thus, we need to use general techniques to obtain truthful mechanisms: maximal in range mechanisms (for deterministic truthfulness) and maximal in distributional range mechanisms (for truthfulness in expectation).

Organization. In Section 2 we present a general overview of literature on game theoretic approaches to pollution control. The reader can skip this section during first reading with no harm to further reading. Section 3 contains definitions, the problem formulation and two applications of our model. In Section 4 we present the hardness results for the pollution game and in Section 5 we present our technique to get a constant approximation algorithm and a truthful in expectation mechanism for planar graphs. We conclude in Section 6 with a few open problems.

2. LITERATURE OVERVIEW

An invaluable source of pollution control regulations comes from the managerial approaches in environment policies. The majority of literature in this field dealt with symmetric information. This problem however shows a fundamental asymmetry between the regulatory bodies and pollutants. The research contributions considering environmental policy with asymmetric information and the diffusion nature of pollution have been limited until recently.

In order to control pollution, an incentive mechanism that is environmentally friendly and resource efficient needs to be designed and deployed by regulatory authorities. However, it is not obvious how to design such a mechanism in presence of asymmetric information; just as Hurwicz [18] put it: the firms know that information will be used by the regulator to design a policy which will affect their profits. Hence, they have an incentive to manipulate reported information in order to influence the content of the policy. In this context, Farell [15] discusses the relevance of the Coase Theorem. This theorem basically asserts that bargaining will lead to an efficient outcome regardless of the initial allocation of property if negotiation and trade in presence of externality are possible and the transaction costs are sufficiently low. Considering the problems of incomplete information, this paper shows that voluntary negotiation does not lead to the first-best outcome that maximizes joint surplus in the presence of two-sided private information. That is to say, centralised economic institutions such as government control and intervention, and decentralised institutions such as bargaining and ownership rights, should be viewed as complementary to each other. Therefore, a necessary condition for the government when designing an optimal pollution control plan is the truthful information about firms.

Kwerel [25], Dasgupta et al. [11] and Spulber [36] have

[1] Polynomial Time Approximation Schemes

proposed mechanisms that implement truth telling by firms to maintain a mild level of pollution. Under the assumption that firms can communicate with the regulator but not with one another, In Kwerel's scheme [25] firms are informed in advance that their messages will be translated into pollution taxes. The regulator issues a fixed number of transferable pollution licences and offers a subsidy for those licences which firms hold in excess of emission. Both the number of licences to be issued and the subsidy rate offered are calculated on the basis of the cost information provided by firms.

Kim and Chang [22] constructed an optimal incentive tax/subsidy scheme in an oligopoly market with pollution and suggested a differential damages mechanism, which leads to an optimal emission level. McKitrick [27] proposes a Cournot Mechanism for pollution control under asymmetric information, in which Nash Equilibrium exists, is stable, and can be reached by iterative computations. Because firms may attempt to manipulate the pollution level allocation to their own advantage, the adjustment rule is exogenous and depends on the actions of the firms. An approach in Karp and Livernois [20] is related to that in Conrad and Wang [10] who examine the steady-state properties of a tax adjustment mechanism in situations where the government has no information about firms' abatement costs.

These prior studies provide an overall framework in the administrative approach to control pollution. However, those models are only a first level of approximation in characterizing the reality. Although, there is some literature studying an economics environment consisting of firms or countries with geographical distinction, few of them take the diffusion nature of air and water pollution into consideration. For instance, Petrosjan and Zaccour [30] study the problem of allocation over time of total cost incurred by countries in a cooperative game of pollution reduction. Segerson [34] develops a general incentive scheme for controlling nonpoint source pollution[2] that considers the diffusion nature, in which rewards for environmental quality above a given standard are combined with penalties for substandard quality. Based on the work of Petrosjan and Zaccour [30], Belitskaya [9] develops an n-person network game model of emission reduction. Dorner et al. [14] create a multi-objective modeling system using Bayesian probability networks to study nonpoint source pollution. Both the work of Belitskaya and Dorner et al. are different from the setting of this paper, in either model assumption or function settings. In addition to these works built on network framework, Dong et al. [13] models the water pollution problem as a cost sharing problem on a tree network. However, none of the literature mentioned above takes into account the role of governments in pollution abatement, more specifically how to make policies assuming information asymmetry. A model that adequately takes both factors into account is what we need to tackle such problems in reality.

Few other papers have studied air pollution in relation to network models. Singh and Datta [35] use artificial neural network method to identify unknown pollution sources in the groundwater. Gianessi et al. [17] analyze the national water pollution control policies. And, finally, Trujillo and Hugh [39] study multi-objective air pollution monitoring network design. These papers use networks in a very different context than ours.

Turning into current practice, emissions trading is a market-based approach used to control pollution by providing economic incentives for achieving reductions in the emissions of pollutants. Various countries have adopted emission trading systems as one of the strategies for mitigating climate-change by addressing international greenhouse-gas emission [38].

A central authority (usually a governmental body) sets a limit or cap on the amount of a pollutant that may be emitted. The limit or cap is allocated and/or sold by the central authority to firms in the form of emissions permits which represent the right to emit or discharge a specific volume of the specified pollutant [37]. Permits (and possibly also derivatives of permits) can then be traded on secondary markets. For example, the European Union Emissions Trading Scheme (EU ETS) trades primarily in European Union Allowances (EUAs), the Californian scheme in California Carbon Allowances, the New Zealand scheme in New Zealand Units and the Australian scheme in Australian Units [2]. Firms are required to hold a number of permits (or allowances or carbon credits) equivalent to their emissions. The total number of permits cannot exceed the cap, limiting total emissions to that level. Firms that need to increase their volume of emissions must buy permits from those who require fewer permits [37, 38]. Currently a simple auction mechanism for selling EUAs is adopted in Europe, see, e.g., [3]. Furthermore in order to limit the automobile pollution, governments use policies of car taxation [19], [16]. A radical transport policy introduced in the UK and first applied in Central London resulted in 19% reduction of CO_2 emissions (see table 2 in [7]).

3. PRELIMINARIES

3.1 Model and applications

We introduce our model in context of our first application where given an area of pollution sources (e.g. factories) each one owned by an agent, the goal of the government as a regulator is to optimize the social welfare while restricting the levels of emitted pollution. More formally, given a weighted digraph $G = (V, E)$, where V is the set of n pollution sources (players, agents) and edge $(u, v) \in E$ represents the fact that u and v are geographic neighbours i.e. $(u, v) \in E$ if and only if the pollution emitted by u affects v (No geometric assumptions are made.) For each $(u, v) \in E$ weight $w_{(u,v)} = w_{uv}$ denotes a discount factor of the pollution discharged by player u affecting its neighbour v. Without loss of generality, we may suppose $w_{uv} \in (0, 1]$, $\forall(u, v) \in E$.

The government sets the total pollution quota discharged to the environment (by the number of pollution sources that remain open) to be $p \geq \sum_{v \in V} x_v$, where $x_v \in \{0, 1\}$ denotes whether pollution source $v \in V$ will be shut down or not. Each agent v has a non-decreasing benefit function $b_v : \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}$, where $b_v(x_v)$ is a concave increasing function (economic diminishing marginal utility phenomenon) with $b_v(0) = 0$ which represents the benefit incurred by v. Each v has a non-decreasing damage function $d_v : \mathbb{R}_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}$, and b_v is concave increasing (the damaging effect of more

[2]Nonpoint source (NPS) pollution refers to both water and air pollution from diffuse sources, that is sources without a specified fixed location. For instance, nonpoint source water pollution affects a water body from sources such as polluted runoff from agricultural areas draining into a river, or wind-borne debris blowing out to sea. Our paper deals mainly with point source pollution.

emitted pollution is accelerating),[3] $b_v(0)=0$ and d_v is convex increasing [25]. Player v's total welfare r_v is v's benefit minus his damage cost: $b_v(x_v)-d_v\left(x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u\right)$, where, $\delta_G^-(v)=\{u\in V:(u,v)\in E\}$, $\delta_G^+(v)=\{u\in V:(v,u)\in E\}$. Thus, player v is affected via the damage function by his own discharged pollution if $x_v=1$ and by the total discounted pollution of his neighbours. This models that pollution spreads along the edges of G. We assume that the government decides on the allowable local level of pollution p_v, for every $v\in V$. This imposes the following constraints for every player $v\in V$: $x_v\le q_v,\ \ x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u\le p_v$. Our first application assumes $x_v\in\{0,1\}$ and $q_v=1$, $\forall v\in V$. In the second application $x_v\in\{0,1,\dots,q_v\}$ and $q_v\in\mathbb{N}$.

The problem of social welfare maximization can be formulated in the general form by the following integer program:

$$\max\ R(x)=\sum_{v\in V}\left(b_v(x_v)-d_v\left(x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u\right)\right)\quad(1)$$

$$\text{s.t.}\qquad \sum_{v\in V}x_v\le p\quad(2)$$

$$x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u\le p_v,\ \ \forall v\in V\quad(3)$$

$$x_v\in\{0,1,\dots,q_v\},\ \ \forall v\in V\quad(4)$$

where (2) is called global constraint and (3) are local constraints. We call $x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u$ the local level of pollution of v. The bound q_v is decided by the government and for this application has value of 1. The above convex integer program is called a pollution game (PG) on G. We call this problem PG with integer variables (if $x_v\in\mathbb{Z}$) or with binary variables (if $x_v\in\{0,1\}$). For an instance I of PG, $|I|$ denotes the number of bits to encode I, and if $q\in poly(|I|)$, where $q=\max_{v\in V}\{q_v\}+1$, we call this problem PG with polynomial size integer variables.

In our second application formulated by the above convex program we consider an area of n cities each one administered by its mayor (agent) where there are available observations of car traffic patterns. More precisely we consider a network represented by a weighted digraph $G=(V,E,w)$, where V is the set of n agents (mayors of the cities), E is the set of roads connecting cities s.t. $(u,v)\in E$ if and only if u and v are neighbouring cities, and $w:E\to\mathbb{R}$ represents the percentage of cars entering a city from a neighboring one i.e. w_{uv} denotes the percentage of cars driving from u to v in some time interval measured by observations. The duty of the regulator is to allocate a number of licences to the agents (mayors) such that the total welfare is maximized while fulfilling a number of constraints. We denote by x_u the number of licences allocated to agent u. The agent with x_u licences gains a benefit of $b_u(x_u)$ which is a monetary income coming from selling these x_u licences to car drivers (our model does not model this explicitly but just assumes for simplicity that all x_u licences are sold). The pollution damage caused in the area of agent u is given by $d_u(x_u+\sum_{v\in\delta_G^-(u)}w_{vu}x_v)$. The total welfare that the regulator aims to maximize is: $\sum_{u\in V}b_u(x_u)-d_u(x_u+\sum_{v\in\delta_G^-(u)}w_{vu}x_v)$.

The total number of licences is bounded by a number p given in the input of the problem. Naturally a percentage of cars with licences from city u remains in u and the rest is split and drives into the neighboring cities. We denote by w_u the percentage of cars remaining in u and w'_{vu} the percentage of cars entering u from neighbouring city v. The maximum number of cars (maximum number of licences) allowed at any moment in city u is bounded by p'_u also given in the input. This is represented by the local constraint: $w_ux_u+\sum_{v\in\delta(u)}w'_{vu}x_v\le p'_u$. If $w_u\neq 0$ the last inequality can equivalently be written as $x_u+\sum_{v\in\delta(u)}w_{vu}x_v\le p_u$, where $w_{vu}=w'_{vu}/w_u$ and $p_u=p'_u/w_u$. There is also a bound q_u on the number of licences issued in city $u,\forall u\in V$, thus we can model this second application as $(1)-(4)$.

Planar graphs are close to real applications, and it is natural to study our second application on planar networks [40]. Imagine a collection of cities (each being a contiguous geographic area) and roads connecting them. This defines a planar map where we only consider edges (roads) between neighbouring cities, which implies a planar graph. We disregard other roads and we consider only frequent driving patterns in a time interval measured by observations. They correspond to frequent commuters, e.g., between house and work, which typically are neighbouring cities.

In the following sections we assume that b_v and d_v are both linear functions with slopes s_v^0 and s_v^1 respectively, i.e. $b_v(x)=s_v^0x$ and $d_v(y)=s_v^1y$, for any $v\in V$. The social welfare function is $R(x)=\sum_{v\in V}\omega_vx_v$, where $\omega_v=s_v^0-s_v^1-\sum_{u\in\delta_G^+(v)}s_u^1w_{vu}$ $(R(x)=\sum_{v\in V}b_v(x_v)-d_v(x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u)=\sum_{v\in V}s_v^0x_v-s_v^1(x_v+\sum_{u\in\delta_G^-(v)}w_{uv}x_u)=\sum_{v\in V}\omega_vx_v)$.

3.2 Basic definitions

Let $I=(G,\mathbf{b},\mathbf{d},\mathbf{p},\mathbf{q})$ be an instance of PG, where $\mathbf{b}=(b_v)_{v\in V}$, $\mathbf{d}=(d_v)_{v\in V}$, $\mathbf{p}=(p_v)_{v\in V}$ and $\mathbf{q}=(q_v)_{v\in V}$ (b_v is assumed private information of v and other parameters are public). Let $\mathcal{I}$ be the set of all instances, and $\mathcal{X}$ the set of feasible allocations. Given a digraph $G=(V,E)$, $G^{un}=(V,E^{un})$, where $E^{un}=\{(u,v):(u,v)\in E\text{ or }(v,u)\in E\}$. A mechanism $\phi=(X,P)$ consists of an allocation $X:\mathcal{I}\to\mathcal{X}$ and payment function $P:\mathcal{I}\to\mathbb{R}^{|V|}_{\ge 0}$ ($X(I)$ satisfies (2)–(4)). For any vector x, x_{-u} denotes vector x without its u-th component. Note, $r_v(X(I))=b_v(X_v(I))-d_v(X_v(I)+\sum_{u\in\delta_G^-(v)}w_{uv}X_u(I))$ is the welfare of player v under $X(I)$. A mechanism $\phi=(X,P)$ is truthful, if for any b_{-v}, b_v and b'_v, $r_v(X(b_v,b_{-v}))-P_v(b_v,b_{-v})\ge r_v(X(b'_v,b_{-v}))-P_v(b'_v,b_{-v})$. A randomized mechanism is truthful in expectation if for any b_{-v}, b_v and b'_v, $\mathbb{E}(r_v(X(b_v,b_{-v}))-P_v(b_v,b_{-v}))\ge\mathbb{E}(r_v(X(b'_v,b_{-v}))-P_v(b'_v,b_{-v}))$, where $\mathbb{E}(\cdot)$ is over the algorithm's random bits. $OPT_G^{fr}(PG)$ ($OPT_G^{in}(PG)$, resp.) denotes the value of the optimal fractional (integral, resp.) solution of PG on G. A mechanism is individually rational if each agent v has non-negative utility when he declares b_v, regardless of the other agents' declarations. The integrality gap of PG on G is defined as $\frac{OPT_G^{fr}(PG)}{OPT_G^{in}(PG)}$. The approximation ratio of an algorithm $\mathcal{A}$ w.r.t. $OPT_G^{in}(PG)$

[3] [25] uses cost function rather than benefit function, which can be viewed as $M_v-b_v(x_v)$, with M_v a large constant for any $v\in V$. The author assumes that cost function is convex decreasing and it is equivalent to $b_v(x_v)$ being a concave increasing function. We use benefit function rather than cost function for ease of analysis.

(resp. $OPT_G^{fr}(PG)$) is $\eta^{in}(\mathcal{A}) = \frac{OPT_G^{in}(PG)}{R(\mathcal{A})}$ ($\eta^{fr}(\mathcal{A}) = \frac{OPT_G^{fr}(PG)}{R(\mathcal{A})}$), where $R(\mathcal{A})$ is the objective value of the $\mathcal{A}$'s solution. If unspecified, the approximation ratio refers to η^{in}. An FPTAS (PTAS, EPTAS, resp.) [4] for a problem $\mathcal{P}$ is an algorithm $\mathcal{A}$ that for any $\epsilon > 0$ and any instance I of $\mathcal{P}$, outputs a solution with the objective value at least $(1-\epsilon)OPT_I^{in}(\mathcal{P})$ and terminates in time $poly(\frac{1}{\epsilon}, |I|)$ ($(\frac{1}{\epsilon}|I|)^{g(\frac{1}{\epsilon})}$ and $g(\frac{1}{\epsilon})poly(|I|)$, resp.), where g is a function independent from I. Let $\gamma_k = \min\{2k^2+2, 8k, \frac{k}{(1-\frac{1}{k}(1+(\frac{2}{k})^{\frac{1}{3}}))^k}\} = (e+o(1))k = O(k)$, and $[n] = \{1, \ldots, n\}$.

Algorithms for packing problems. A linear constraint $ax \leq b$ with $x \in \mathbb{N}_{\geq 0}^n$ an integer vector, and $a, b \in \mathbb{R}_{\geq 0}^n$ are vectors, is called a packing constraint. A linear (resp. convex) maximization programming problem with packing constraints is called a linear (resp. convex) packing programming problem. A k-column-sparse packing integer program is one in which each variable j participates in at most k constraints.

Proposition 1 ([8],[31]). *There is a polynomial time deterministic algorithm for k column sparse linear packing programming problem with binary variables, achieving the approximation ratio $\eta^{fr} = \gamma_k$.*

Proposition 2 ([26]). *For any linear packing programming problem, if there is a polynomial deterministic algorithm with the approximation ratio η^{fr} for this problem, then there is a polynomial, randomized, individually rational, η^{fr}-approximation mechanism for the same problem that is truthful in expectation.*

4. HARDNESS

PG is weakly NP-hard even on trees without global constraint. Suppose that the underlying graph is a star. We observe that we can reduce to PG the knapsack problem, where the weights w_{uv} are the sizes of the items.

We also note that inequality (2) can also be written as equality

$$\sum_{u \in V} x_u = p \tag{$2'$}$$

since the upper limit of the total amount of the pollution is controlled by the government. If $\sum_{v \in V} x_v < p$ in the final allocation, then the government can simply set p equal to $\sum_{v \in V} x_v$ without any changes. However, for computational issues, these two representations lead to different computational complexity. If inequality (2) is replaced by ($2'$), then even finding a feasible solution of PG is NP-complete. Therefore, unless stated otherwise, we always suppose inequality (2) as a constraint of PG.

Theorem 1. *Finding a feasible solution which satisfies constraints ($2'$) and (3) to PG when $p_v = 1\ \forall v \in V$ and $w_{uv} > 0$ for any $(u,v) \in E$ is NP-complete.*

Proof. It is straight forward that the problem is in NP. Consider now a formula of monotone 1-in-3 SAT where an instance of this problem consists of n Boolean variables and m clauses. A YES instance is one in which an assignment to its Boolean variables is such that exactly one literal from each clause is true. The problem is known to be NP-Complete even when there are no negations [33]. The following proof is inspired by the reduction of 3-SAT to Independent Set (p. 248 [12]).

Let us represent a clause, say $(x \vee y \vee z)$, by a triangle with vertices labeled x, y, z. Repeat this construction for all clauses. Next consider one of the literals, say x, which appears in k clauses $C_{i_1}, \ldots, C_{i_k}$. Let $Tr_{i_1}, \ldots, Tr_{i_k}$ be the triangles of the clauses $C_{i_1}, \ldots, C_{i_k}$ respectively. Then connect x of Tr_{i_1} with all the vertices of $Tr_{i_2}, \ldots, Tr_{i_k}$ except those labeled with x. Repeat this construction for x in all these triangles and for all literals. For example consider the formula $\Phi = (x_1 \vee x_2 \vee x_3)(x_1 \vee x_4 \vee x_5)$ (see Figure 1). First construct the triangles labeled x_1, x_2, x_3 and x_1, x_4, x_5 for the two clauses respectively. Then connect vertex x_1 of the first clause with the vertices x_4 and x_5 of the second clause. In the same way connect vertex x_1 of the second clause with the vertices x_2 and x_3 of the first clause.

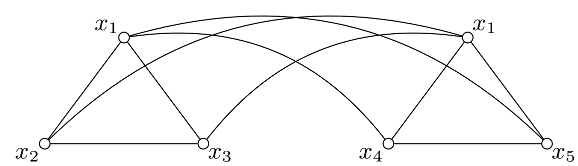

Figure 1: A gadget of reduction

Consider now an instance of 1-in-3 SAT which is true and let $G = (V, E)$ be the corresponding graph constructed as above. Furthermore for every vertex $u \in V$, let $p_u = 1$ and $p = m$. Suppose that we have a truth assignment which satisfies all the clauses. Then this means we choose p vertices in G without violating any of the constraints. Indeed if two vertices have the same label then they are not connected. If they have different labels, say x from clause C_1 and y from clause C_2 and they are connected, this is because their corresponding clauses have a common literal, either x or y. Thus if one of them has value true, the other will have value false for the formula to be satisfiable. Finally if two vertices belong to the same clause, only one of them will have the value true.

Suppose now that we can decide in polynomial time whether there is a solution in an instance of a graph constructed by a formula as described above with $p = m$ vertices when $p_v = 1$, $\forall v \in V$. Then setting the literal in the set $\{v : x_v = 1\}$ is a solution of 1-in-3 SAT. The argument is as follows, in each triangle, there is exactly one vertex such that its value is one since at most one vertex in each triangle can be selected and there are m triangles and $p = m$. By the construction of G, these vertices consist of a solution of 1-in-3 SAT. Thus exactly one literal in every clause has the value true in the formula. $\square$

Theorem 2. *There is no EPTAS for PG with binary variables on the directed planar graph $G = (V, E)$ when b_v and d_v are both linear functions, for any $v \in V$.*

Proof. Consider PG on the following simple planar graph. There are $n+2$ vertices labeled as $\{o_1, o_2, 1, 2, ..., n\}$ and the edge set $E = \{(i, o_j), i \in [n], j \in [2]\}$. Weights w_{io_j},

[4] Fully Polynomial Time Approximation Scheme, Polynomial Time Approximation Scheme and Efficient Polynomial Time Approximation Scheme respectively

$i \in [n], j \in [2]$. For any two dimensional knapsack problem, there exists an instance of PG with binary variables without the global constraint on such a simple graph exactly corresponding to this two dimensional knapsack problem. According to [24], there is no EPTAS for two dimensional knapsack. Hence, there is no EPTAS for PG on this simple planar graph. □

5. CONSTANT APPROXIMATION ON PLANAR GRAPHS

5.1 Algorithms for (α, β)-decomposition

Given a digraph $G = (V, E)$ and a subset $U \subset V$, we call significant neighbours of U, $SN_G(U)$, all the vertices in $V \backslash U$ with at least two neighbours in U (see figure 2). Consider a partition $\{V^i\}_{i=1}^{\alpha}$ of V. Now let $SN_{G^{un}}(V^i) = \{u \notin V^i \mid \exists v \in V^i, \text{ s.t. } u \text{ is a significant neighbour of } v \text{ w.r.t. } V^i\}$ denote the significant neighbours of V^i in G^{un}. Let G^i be the induced subgraph of $V^i \cup SN_{G^{un}}(V^i)$ in G^{un}. A partition $\{V^i\}_{i=1}^{\alpha}$ of V is called an (α, β)-partition (or (α, β)-decomposition) of G if for any $i \in [\alpha]$ and $v \in V^i$, $|\delta_{G^i(v)}| \leq \beta$, where α, β are two given positive integers.

According to the following Lemma 1, we can obtain a constant approximation for PG with integer variables for any graph with (α, β)-decomposition. Such a decomposition of planar graphs will be presented later.

LEMMA 1. *If a directed graph G has an (α, β)-decomposition, then there is a deterministic $(\eta^{fr} = \alpha\gamma_{\beta+2} + 1)$-approximation algorithm for PG with integer variables, and, a truthful in expectation mechanism for the same problem with the same approximation.*

PROOF. If there is an η^{fr}-approximation algorithm for a linear packing problem with binary variables, then there is an $(\eta^{fr} + 1)$-approximation algorithm for the same problem with integer variables [8]. Hence, it is sufficient to show that there is an $(\eta^{fr} = \alpha\gamma_{\beta+2})$-approximation algorithm for PG with binary variables. Now we consider PG with binary variables. Let $\{V^i\}_{i=1}^{\alpha}$ be an (α, β)-decomposition of graph G. Let x^* be the optimal fractional solution of PG with binary variables. Then $R(x^*) \leq \alpha \max_{i \in [\alpha]}\{R(x^*_{V^i})\}$, where $x^*_{V^i}$ is a fractional solution such that its value is equal to x^*_v, for any $v \in V^i$ and 0 otherwise. Let PG_i denote the PG on G by setting $x_v = 0$, for any $v \notin V^i$. Note that $x^*_{V^i}$ is a feasible solution for PG_i, which gives $R(x^*_{V^i}) \leq OPT^{fr}(PG_i)$. W.l.o.g. we suppose $w_{uv} \leq p_v$, for any $(u, v) \in E$ and $v \in V$ (otherwise $x_v \equiv 0$ for PG). Observe that in PG_i, only x_v, $v \in V^i$ are variables. Now for any $v \in V^i$, let us see how many constraints in PG_i contain x_v. Suppose $u \in V \backslash V^i$ is a neighbour of v in G^{un}. If u is not a significant neighbour of v, since $w_{vu} \leq p_u$, we can remove the constraint $w_{vu}x_v \leq p_v$ in PG_i. Hence, only the local constraints of the significant neighbours of v remain containing variable x_v. As $\{V^i\}_{i=1}^{\alpha}$ is an (α, β)-decomposition of graph G, there are at most $\beta + 1$ local constraints containing variable x_v (which includes the local constraint of vertex v itself). Together with the global constraint, we know x_v appears in at most $\beta + 2$ constraints in PG_i, for any $v \in V^i$, which means PG_i is $\beta + 2$ column sparse. Therefore, by Proposition 1, there is a polynomial deterministic algorithm for PG_i with binary variables, finding an integer solution y_i for PG_i such that $\gamma_{\beta+2}R(y_i) \geq OPT^{fr}(PG_i)$, for any $i \in [\alpha]$. Then $\alpha\gamma_{\beta+2} \max_{i \in [\alpha]}\{R(y_i)\} \geq \alpha \max_{i \in [\alpha]}\{OPT^{fr}(PG_i)\} \geq \alpha \max_{i \in [\alpha]}\{R(x^*_{V^i})\} \geq R(x^*) = OPT^{fr}(PG)$. A truthful in expectation mechanism with the same approximation ratio is guaranteed by Proposition 2. □

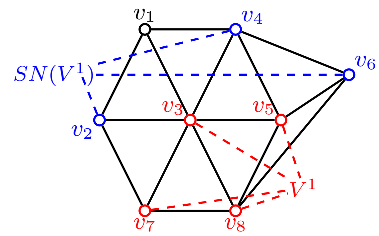

Figure 2: Significant neighbours $SN(V^1) = \{v_2, v_4, v_6\}$ of $V^1 = \{v_3, v_5, v_7, v_8\}$ in the graph of solid black lines.

Planar graphs. The integrality gap of PG on planar graphs is at least 4 as shown by a complete graph with four vertices. For a small $\epsilon > 0$, let $w_{uv} = \epsilon$, for any $(u, v) \in E$, and $p_v = \omega_v = 1$, for any $v \in V$. There is no global constraint. The optimal integer solution of PG on this graph is $x_v = 1$ for some $v \in V$ and $x_u = 0$ for all $u \neq v$, implying the optimal objective value 1. However, setting $x_v = 1 - 4\epsilon$, for any $v \in V$ provides a feasible fractional solution, which gives the objective value $4 - 16\epsilon$. Therefore, the integrality gap is at least 4, meaning that our LP relaxation cannot lead to better than 4 (e.g., PTAS) approximations.

We provide an (α, β)-decomposition of any planar graph, with $\alpha = 18$, $\beta = 6$ or $\alpha = 54$, $\beta = 4$. We did not attempt to optimize these two parameters.

THEOREM 3. *There is an (α, β)-decomposition of a directed planar graph $G = (V, E)$, where $(\alpha, \beta) = (18, 6)$ or $(54, 4)$.*

PROOF. Let $G' = G^{un}$. Suppose G' is connected, otherwise we can run the algorithm on each connected component respectively. Define the sequence of vertex sets $\{N_i\}_i$ of G' as follows. Fix an arbitrary vertex $v_0 \in V$, and let $N_1 = \{v_0\}$, and N_i is defined recursively as

$$N_{i+1} = \{v \in V \backslash \bigcup_{j=1}^{i} N_j \mid (v, u) \in G', \text{ for some } u \in N_i\},$$

for $i = 1, 2, ..., |V|$. By this definition, for any $v \in N_i$ and $u \in N_j$, if $|i - j| \geq 2$, then $(u, v) \notin E'$. We also observe that N_i is the set of vertices with distance $i - 1$ to v_0 in G' (i.e., the shortest path distance with respect to the number of edges). Suppose the length of the sequence $\{N_i\}_i$ is K. Let $S_i = \{j \equiv i \,(\mathrm{mod}\, 3) \mid j \in [K]\}$, $i \in [3]$. Let also $S_0 = S_3$, and $V^i = \bigcup_{j \in S_i} N_j$, $i \in [3]$. We will now need the following Lemmas 2, 3 and 4.

LEMMA 2. *For each $v \in N_j$, the number of significant neighbours of v in N_{j-1} w.r.t. N_j is at most two.*

PROOF. Suppose there exists $v_1 \neq v_2 \neq v_3 \in N_{j-1} \cap \delta_{G'}(v)$ and $v \neq u_1, u_2, u_3 \in N_j$ such that $(u_i, v_i) \in G'$, $i \in [3]$ (see Figure 3). By the definition of N_j, there is a path from v_0 to v_i, $i \in [3]$, and $(v, v_i) \in G'$, $i \in [3]$. W.l.o.g. suppose v_2 is inside the circle constructed from the path of v_0 to v_1, v_3 and edges (v, v_1) and (v, v_3) in the planar embedding. Then (u_2, v_2) will intersect this circle, which contradicts that G' is planar. □

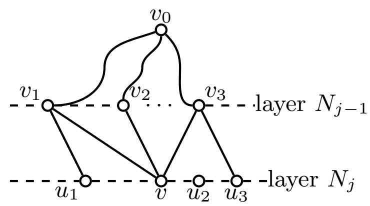

Figure 3: An illustration of relations between N_j and N_{j-1}.

Next, we partition N_j into two sets N_j^1 and N_j^2 such that each vertex in N_j^i has at most two significant neighbours in N_{j+1} w.r.t. N_j^i, $i \in [2]$. We say two vertices $v, u \in N_j$ are connected by a zigzag path if there exists a path $(v, v_1, v_2, v_3, .., v_s, u)$ in G' such that v_i and v_{i+1} alternatively belong to N_{j+1} and N_j, i.e., $v_1 \in N_{j+1}$ and $v_2 \in N_j$. Note that s must be odd. We define the zigzag length of this zigzag path as $\frac{s+1}{2}$. The zigzag distance between v and u, denoted d^z_{uv}, is defined as the zigzag length of the shortest zigzag path between v and u if there exists one and ∞ otherwise. Note that the zigzag distance of v to itself is zero. The partition algorithm PA_1 works as follows (see Algorithm 1). Let $N_j^1 = A_1$ and $N_j^2 = A_2$, where A_1, A_2 is output of PA_1. (Note that PA_1 is run for each $j \in [K]$.)

Algorithm 1: (PA_1)

Input: $A_1, A_2 \leftarrow \emptyset$, $B \leftarrow N_j$
Output: A_1, A_2
while $B \neq \emptyset$ **do**
  Select a vertex $v \in B$;
  Find B_1, B_2 below by BFS.
  $B_1 \leftarrow \{u \in N_j \mid d^z_{uv} \text{ is odd}\}$;
  $B_2 \leftarrow \{u \in N_j \mid d^z_{uv} \text{ is even}\}$;
  $A_1 \leftarrow A_1 \cup B_1$; $A_2 \leftarrow A_2 \cup B_2$;
  $B \leftarrow B \backslash (B_1 \cup B_2)$;

Algorithm 2: (PA_2)

Input: $A_1, A_2, A_3 \leftarrow \emptyset$, $B \leftarrow N_j^i$
Output: A_1, A_2, A_3
while $B \neq \emptyset$ **do**
  Select a vertex $v \in B$;
  Find B_k's below by BFS.
  for $k \leftarrow 1$ **to** 3 **do**
    $B_k \leftarrow \{u \in N_j^i \mid d^{N_j}_{uv} \equiv k (\mathrm{mod}\ 3)\}$
    $A_k \leftarrow A_k \cup B_k$;
  $B \leftarrow B \backslash (B_1 \cup B_2 \cup B_3)$;

Lemma 3. *For each $v \in N_j^i$, v has at most two significant neighbours in N_{j+1} w.r.t. N_j^i, $i \in [2]$.*

Proof. First, note that if v and u are selected in different iterations of the while loop in Algorithm 1, there is no zigzag path between them. Therefore, for a single iteration of the while loop, suppose $v \in B$ is selected. We only need to show that for any $u \in B_i$, u has at most two significant neighbours in N_{j+1} w.r.t. B_i, $i \in [2]$. First, note that $v \in B_2$ (its zigzag distance to itself is 0). Since all the other

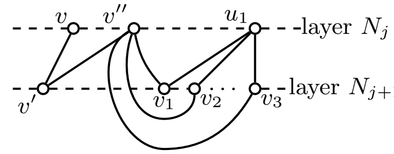

Figure 4: Relations between N_j and N_{j+1}

vertices in B_2 have zigzag distance to v at least two, v has no significant neighbours w.r.t. B_2 in N_{j+1}. Now fix $i \in [2]$. Consider any two vertices $u_1, u_2 \in B_i$, u_1 and u_2 connect to the same vertex in N_{j+1} only if they have the same zigzag distance to v. Suppose there exists three different vertices v_1, v_2, $v_3 \in N_{j+1}$, such that they are significant neighbours of u_1 w.r.t. B_i (see Fig. 4). By similar arguments as above, there exists zigzag paths from v to v_i, $i \in [3]$. Also note that edges $(u_1, v_i) \in G'$, $i \in [3]$. W.l.o.g. suppose v_2 is in the circle constructed from the zigzag paths v to v_1, v_3 and edges (u_1, v_1) and (u_1, v_3). Since the graph G' is planar, there exists no edge between v_2 and another vertex in B_i with the same zigzag distance as u_1. Therefore, u_1 has at most two significant neighbours w.r.t. B_i in N_{j+1}. □

Next we will partition each set N_j^i, $i \in [2]$, $j \in [K]$ into a constant number of sets $\{N_j^{ik}\}_k$ such that each vertex in N_j^{ik} has at most a constant number of significant neighbours w.r.t. N_j^{ik} in N_j. We provide two partition algorithms. Both algorithms in spirit are similar to Algorithm 1. For any two vertices v, $u \in N_j^i$, we say they are connected by a N_j-path if there exists a path $(v, v_1, v_2, \cdots, v_s, u)$ in G' such that $v_\ell \in N_j$, $\forall \ell \in [s]$. N_j-distance of two vertices v, $u \in N_j^i$, denoted $d^{N_j}_{uv}$, is defined as the number of edges of the shortest N_j-path between v and u if there exists one and ∞ otherwise. The process works as PA_2 (Algorithm 2). Note that $v \in B_3$, because the N_j-distance from v to itself is zero. Let $N_j^{ik} = A_k$, $k \in [3]$ (where A_1, A_2, A_3 are a partition of N_j^i output by PA_2).

Lemma 4. *For any $k \in [3]$, and each $v \in N_j^{ik}$, v has at most 2 neighbours in N_j, or has no neighbours in N_j^{ik} nor significant neighbours w.r.t. N_j^{ik} in $N_j \backslash N_j^{ik}$.*

Proof. First, note that if v and u are selected in different iterations of while loop in Algorithm 2, there is no N_j-path between them. Therefore, for a single iteration of the while loop, suppose $v \in B$ is selected. Since $v \in B_3$ (N_j distance to itself is 0), v has no neighbours in B_3 nor significant neighbours w.r.t. B_3 in $N_j \backslash B_3$ by PA_2. Now fix $k \in [3]$. Consider any two vertices $u_1, u_2 \in B_k$, u_1 and u_2 connect to the same vertex in N_j only if they have the same N_j-distance to v. Next we will show for any $u_1 \neq v$ and $u_1 \in B_k$, for any k, u_1 has at most two neighbours in N_j. Suppose there exist three different vertices u_1, u_2, $u_3 \in N_j$, such that $(u_1, u_2) \in G'$ and $(u_1, u_3) \in G'$. By similar arguments as above, there exists N_j-paths from v to u_i, $i \in [3]$. Since $u_i \in N_j$, $i \in [3]$, there exist paths in G' from v_0 to u_i, $i \in [3]$. We observe that it is only possible that u_1 is in the circle constructed from the N_j paths v to u_2, u_3 and paths from v_0 to u_2 and u_3 (the case where u_2 or u_3 is in the circle constructed by the other two vertices with v and v_0 will violate the planarity of G') (see Fig. 5). Since graph G' is planar, there exists no edge between u_1 and another vertex

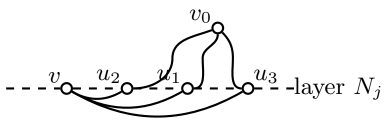

Figure 5: Relations between N_j and N_j.

in N_j (due to that such a vertex will have a path to v and v_0 respectively). Therefore, u_1 has at most two neighbours in N_j. $\square$

Combining Lemmas 2, 3 and 4, $\{N_j^{ik}\}_{ijk}$ is an (α, β)- decomposition of G with $(\alpha, \beta) = (18, 6)$. The second partition procedure is based on Algorithm 2 and we just sketch it here. We can partition N_j^{ik} into another three sets, e.g. $N_j^{ik\ell}$, $\ell \in [3]$ by the same arguments as above by just partitioning it according to the length of path (mod 3) such that each vertex in $N_j^{ik\ell}$ will not have any neighbours in $N_j^{ik\ell}$. This procedure is exactly the same as Algorithm 2 where N_j^i is replaced with N_j^{ik}. Thus $\{N_j^{ik\ell}\}_{ijk\ell}$ is a $(54, 4)$-decomposition of G, which finishes the proof of Theorem 3.

By Theorem 3 and Lemma 1, and observing that $18 \min\{\gamma_8, 3\gamma_6\} = 18\gamma_8 = O(1)$, we have

THEOREM 4. *There is a randomized, individually rational and truthful in expectation* $(18\gamma_8 + 1)$*-approximation mechanism for PG on planar graphs with integer variables.*

5.2 Better approximation under some mild condition

We will use the 4-color theorem for planar graphs to present an improved $(6 + \epsilon)$-approximate truthful in expectation mechanism for PG under the following natural (and mild) assumption:

$$\sum_{u \in \delta_G^-(v)} w_{uv} \leq p_v \tag{5}$$

This constraint means that if each of v's neighbours emits only one unit amount of pollution, the level of pollution in v will not exceed v's local level of pollution. Let x^1 be the optimal fractional solution of PG with binary variables without global constraint on planar graph G.

THEOREM 5. *Suppose condition (5) holds and* $R(x^1) \geq 1$. *There is a randomized, individually rational,* $(\eta^{fr} = 6 + \epsilon)$*-approximation mechanism that is truthful in expectation for PG on planar graphs with integer variables, terminating in time* $poly(|I|, \log(\frac{1}{\epsilon}))$.

PROOF. Note that if condition (5) holds, then every independent set is a feasible solution for PG with binary variables without global constraint. By 4-color theorem [5, 32] for planar graphs, there is an independent set $S \subset V$ such that $4R(z_S) \geq R(x^1)$ where z_S is defined by $z_v = 1$ if $v \in S$ and $z_v = 0$ otherwise. Further there is an $O(|V|^2)$ algorithm finding z_S [32]. By theorem 3 of [23] and $R(x^1) \geq 1$, there is a deterministic $(\eta^{fr} = 5 + \epsilon)$-approximation algorithm for PG with binary variables, running in $poly(|I|, \log(\frac{1}{\epsilon}))$ time. Then there is a deterministic $(\eta^{fr} = 6 + \epsilon)$-approximation algorithm for PG with integer variables, running in time $poly(|I|, \log(\frac{1}{\epsilon}))$ [8]. By Proposition 2, this $(\eta^{fr} = 6 + \epsilon)$-approximation mechanism is truthful in expectation for PG with integer variables. $\square$

6. CONCLUSION AND DISCUSSION

We present a new network model for the pollution control problem which is studied on planar networks. These networks can be applied to model air pollution from diffuse sources. Our results include computational hardness results, a constant approximation algorithm and truthful in expectation mechanisms. We obtain these results by introducing novel algorithmic techniques for planar graphs which are of independent interest. Many interesting open problems arise:

1. Determine whether PG with binary variables on planar graphs admits a PTAS or is APX-hard.

2. What about lower bounds on truthful (deterministic, universal, truthful in expectation) mechanisms for PG? Can externality be used to obtain such lower bounds?

3. How to generalize our results to other graphs, e.g., Euclidean graphs?

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